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**1.** Prove, from first principles, that the derivative of $3x^2$ is $6x$.

**(4)**

Let $f(x) = 3x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + hx + hx + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6hx + 3h^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{6hx + 3h^2}{h}$$

$$= \lim_{h \to 0} 6x + 3h$$

As $h \to 0$, $3h \to 0$, so $f'(x) \to 6x$

So in the limit, $\boxed{\text{derivative} = 6x}$

**(Total for Question is 4 marks)**

**2.** Prove, from first principles, that the derivative of $x^3$ is $3x^2$

**(4)**

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2$$

gradient of chord $= 3x^2 + 3xh + h^2$

when $h \to 0$, $3xh \to 0$ $h^2 \to 0$

gradient of curve $= 3x^2$

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**3.** Given that $\theta$ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

**(5)**

Definition of the derivative : $\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$ 1

$\Rightarrow f(\theta) = \sin\theta \quad , \; f(\theta+h) = \sin(\theta+h)$

Compound Angle Identity : $\sin(\theta+h) = \sin\theta\cos(h) + \sin(h)\cos\theta$ 1

$\Rightarrow f'(\theta) = \lim_{h\to 0} \frac{\sin\theta\cos(h) + \sin(h)\cos\theta - \sin\theta}{h}$ 1

$\Rightarrow f'(\theta) = \lim_{h\to 0} \sin\theta\frac{(\cos(h)-1)}{h} + \frac{\sin(h)}{h}\cdot\cos\theta$ 1

$\boxed{\frac{\cos(h)-1}{h} \to 0 \qquad \frac{\sin(h)}{h} \to 1}$

$\Rightarrow f'(\theta) = \sin\theta \cdot 0 + 1\cdot\cos\theta = \cos\theta$

$\Rightarrow \underline{f'(\theta) = \cos\theta}$ as required.

**(Total for Question is 5 marks)**

**4.** Given that $\theta$ is measured in radians, prove, from first principles, that

$f(\theta) = \cos(\theta)$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

**(5)**

$f(x) = y$

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$\cos(\theta + h) \equiv \cos(\theta)\cos(h) - \sin(\theta)\sin(h)$ ✓1

$$f'(\theta) = \lim_{h \to 0}\left(\frac{\cos(\theta + h) - \cos(\theta)}{h}\right) ✓1$$

$$= \lim_{h \to 0}\left(\frac{\cos(\theta)\cos(h) - \sin(\theta)\sin(h) - \cos(\theta)}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{\cos(\theta)\cos(h) - \cos(\theta)}{h}\right) + \lim_{h \to 0}\left(\frac{-\sin(\theta)\sin(h)}{h}\right)$$

$$= \cos(\theta)\left[\lim_{h \to 0}\left(\underbrace{\frac{\cos(h) - 1}{h}}_{0}\right)\right] - \sin(\theta)\left[\lim_{h \to 0}\left(\underbrace{\frac{\sin(h)}{h}}_{1}\right)\right] ✓1$$

$$= \underline{\underline{-\sin(\theta)}} ✓1$$

**5.**

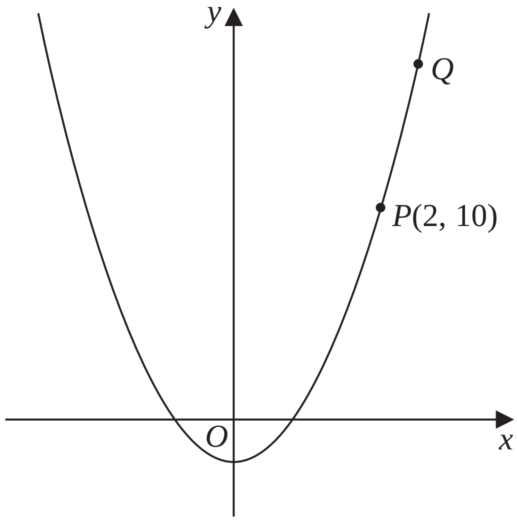


**Figure 1**

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at $P$.

**(2)**

The point $Q$ with $x$ coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line $PQ$, giving your answer in terms of $h$ in simplest form.

**(3)**

(c) Explain briefly the relationship between part (b) and the answer to part (a).

**(1)**

(a) $y = 3x^2 - 2$

$\frac{dy}{dx} = 6x$ (1)

gradient at $P(2, 10) = 6(2) = 12$ (1)

**Question 5 continued**

(b) First we find coordinates of Q in terms of h

$x$ coordinate is given : $2+h$

Find y coordinate : $y = 3x^2 - 2$

$y = 3(2+h)^2 - 2$

$= 3(4+4h+h^2) - 2$

$= 12 + 12h + 3h^2 - 2$

$= 3h^2 + 12h + 10$

Find gradient of PQ $= \dfrac{y_1 - y_2}{x_1 - x_2}$

$= \dfrac{(3h^2+12h+10) - 10}{(2+h) - 2}$ (1)

$= \dfrac{3h^2 + 12h}{h}$ (1)

$= 3h + 12$ (1)

(c) $h \to 0$ , $3h + 12 \to 12$

$\therefore$ The gradient of the chord tends to the gradient of the tangent to the curve. (1)

**(Total for Question 5 is 6 marks)**